

CHARGED PARTICLE IONIZATION AND RANGE

Unlike the neutral radiations (*e.g.*, neutrons and gamma/X rays), the charged particles (*e.g.*, electrons, protons and alphas) are subjected to the coulombic forces from electrons and nuclei within the material through which they pass. Some of the various empirical relationships for charged particle ranges have been developed over time are presented herein.

As a charged particle transverses a material, the particle slows as it loses kinetic energy. The loss of energy by charged particles traveling through a material is broken into two components based on the mechanism of energy transfer—either collisional or radiative energy loss. The total *stopping power* is

$$\frac{dE}{dx} = \left(\frac{dE}{dx} \right)_{col} + \left(\frac{dE}{dx} \right)_{rad} \quad (1)$$

where $(dE/dx)_{col}$ is the electronic energy loss due to Coulomb interactions (*i.e.*, the ionization and excitation), and $(dE/dx)_{rad}$ is the nuclear energy loss (*e.g.*, due to emission of Bremsstrahlung or Cerenkov radiation, and nuclear interactions). *Excitation* raises an electron to a higher energy shell, whereas *ionization* completely removes the electron from the atom. Ionization creates an *ion pair*, which is the (now) free electron and the positively charged atom from which the electron was removed. The freed electron may possess sufficient kinetic energy to cause further ionization events (such energetic electrons are sometimes called *delta rays*).

The term $(dE/dx)_{col}$ is also called the *linear energy transfer* (LET), *i.e.*, the linear rate of energy loss (dE/dx) of a charged particle due to ionization and excitation.

$$LET = \frac{dE}{dx} \quad (2)$$

Some scientists define the LET in terms of dE/dx while others prefer to define the LET with respect to the material density (ρ) such that the LET units are, for example, MeV·cm²/mg. The *mass stopping power* (S) is the total energy lost per path length by a charged particle [MeV/g/cm²]

$$S = \frac{dE/dx}{\rho} \quad (3)$$

Another quantity used to describe particle energy loss is specific ionization. The *specific ionization* is the number of ion pairs (i.p.) formed per unit distance traveled by the charged particle [i.p./cm]:

$$S.I. = \frac{dE/dx \text{ [eV/cm]}}{w \text{ [eV/i.p.]}} \quad (4)$$

where w is the average energy expended to create an ion or electron-hole pair. The energy required to create an electron-hole pair depends on the irradiated material. Experimental results have shown that the actual energy necessary to create an ion pair is about 2-3 times greater than the ionization potential. For instance, the ionization potentials of oxygen and nitrogen are 13.6 and 14.5 eV, respectively, whereas the mean energy to create an ion pair in air is 34 eV.

Material	Air	silicon (Si)	germanium (Ge)	gallium-arsenide (GaAs)	silicon dioxide (SiO ₂)
Ion pair generation energy (w)	34 eV	3.6 eV	2.8 eV	4.8 eV	17 eV

Example:

What is the charge transfer rate in pC/micron for a 37 MeV·cm²/mg particle traveling through silicon?

Solution:

First, obtain a conversion factor between charge and energy. The amount of energy required to create an ion pair is material dependent. Silicon requires 3.6 eV to create each electron-hole pair, so

$$\frac{w_{ehp}}{q} = \frac{3.6 \times 10^{-6} \text{ MeV/ion pair}}{1.6 \times 10^{-7} \text{ pC/ion pair}} = 22.5 \frac{\text{MeV}}{\text{pC}}$$

Next, we can find the equivalent charge transfer rate given the density of silicon ($\rho = 2.33 \text{ g/cm}^3$)

$$\frac{LET \rho}{w_{ehp} / q} = \frac{(37 \text{ MeV} \cdot \text{cm}^2/\text{mg})(2330 \text{ mg/cm}^3)}{(22.5 \text{ MeV/pC})(10,000 \mu\text{m/cm})} = 0.38 \frac{\text{pC}}{\mu\text{m}}$$

Universally for silicon, $1 \text{ pC}/\mu\text{m} = 97 \text{ MeV}/(\text{mg}/\text{cm}^2)$.

Light Charged Particles (e.g., electrons, betas)

For our purposes, light charged particles are electrons, positrons and beta particles. The terminology beta particle is intended to denote an electron originating from a radioactive decay process. When electrons pass through matter, several possible processes may occur:

1. Ionization and excitation in which the energy loss mechanism is similar to that for heavy charged particles;
2. Bremsstrahlung, which is the creation of X-rays from electron deceleration;
3. Elastic scattering from nuclear and electronic interactions; and
4. Nuclear excitation which is usually negligible.

A single expression describing the radiative energy loss by electrons for a large range of kinetic energies is not available. The collisional energy loss (LET) for fast electrons may be computed from the Bethe formula. Electrons (betas) are easily scattered due to their small mass and charge. The electrons travel a nonlinear path and their range in air is on the order of meters.

Beta Particle Range

The maximum range, R_{\max} , (material independent) of a beta particle can be computed from an empirical formula given by Katz and Penfold:[†]

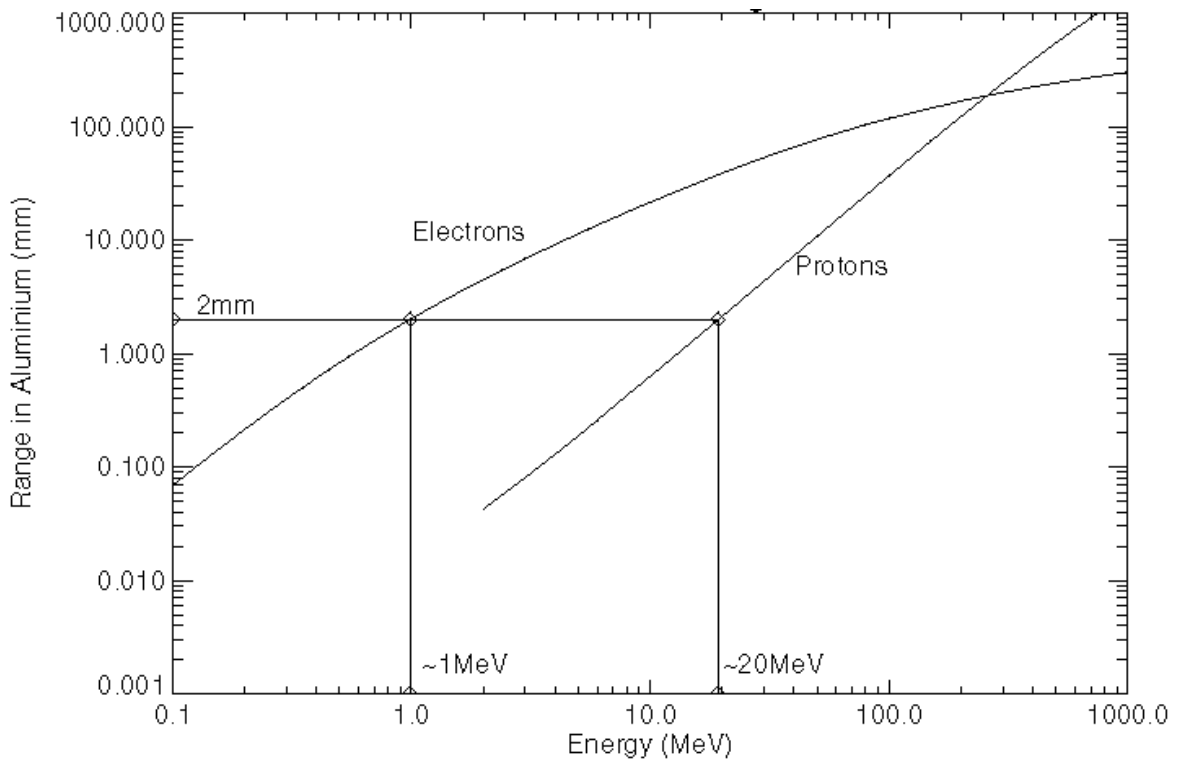
$$R_{\max} [\text{g}/\text{cm}^2] = \begin{cases} 0.412 E_{\beta}^{1.265-0.0954 \ln(E_{\beta})} & 0.01 \leq E_{\beta} \leq 2.5 \text{ MeV} \\ 0.530 E_{\beta} - 0.106 & E_{\beta} > 2.5 \text{ MeV} \end{cases} \quad (5)$$

where E_{β} is the maximum beta energy in MeV. The ability to stop betas depends primarily on the number of electrons in the absorber (*i.e.*, the areal density, which is the number of electrons per cm^2). Hence, the range when expressed as a density thickness (g/cm^2) of the material gives a generic quantifier by which various absorbers can be compared. With the maximum range known, the actual shielding thickness required can be computed

$$t = \frac{R}{\rho} \quad (6)$$

where ρ is the material density. Such an approach can be used to produce energy-range curves. A useful rule of thumb is that the range of a beta particle in g/cm^2 is approximately half its energy in MeV. Beta particles from natural radionuclides are easily stopped by thin sheet of metal or glass. Oftentimes, aluminum is used to shield against protons and electrons. For example, the figure below shows that 2 mm of aluminum is sufficient to shield against 1 MeV electrons and 20 MeV protons.

[†] L. Katz and A.S. Penfold, *Rev. Mod. Phys.*, 24 (1952), p. 28.



Mean ranges of protons and electrons in aluminum [Source: E.J. Daly, A.Hilgers, G. Drolshagen, and H.D.R. Evans, "Space Environment Analysis: Experience and Trends," ESA 1996 *Symposium on Environment Modelling for Space-based Applications*, Sept. 18-20, 1996, ESTEC, Noordwijk, The Netherlands.]

Example:

Determine the copper thickness necessary to stop the beta particles emitted from Co-60.

Solution:

Cobalt-60 emits beta particles with a maximum energy of 0.3179 MeV, hence the maximum range of those particles is

$$R_{\max} = (0.412) (0.3179)^{1.265 - 0.0954 \ln(0.3179)} = 0.08529 \text{ g/cm}^2$$

Copper has a density of 8.933 g/cm³, so the actual shield thickness necessary is

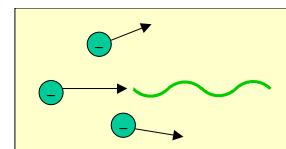
$$\text{thickness} = \frac{R_{\max}}{\rho} = \frac{0.08529 \text{ g/cm}^2}{8.933 \text{ g/cm}^3} = 0.00955 \text{ cm}$$

Electron Range

The average range of an electron from a monoenergetic beam of energy equal to $E_{\beta, \max}$ is greater than that of a beta particle because beta particles have a continuous energy spectrum from zero energy up to a maximum kinetic energy; in particular, the average beta energy is about one-third of the maximum beta energy ($E_{\beta, \text{avg}} \approx \frac{1}{3} E_{\beta, \max}$). To compute the electron range using the Katz-Penfold relation of Eq. (5), we utilize the kinetic energy of the monoenergetic electrons to find the range.

Bremsstrahlung

Bremsstrahlung—which is German for braking radiation—is secondary photon radiation produced by the deceleration of charged particles passing through matter. Bremsstrahlung is primarily electron braking radiation. Hence, beta rays and electrons can give rise to highly penetrating X-rays. When the negatively-charged electron passes near the nucleus, the high-speed electron is



rapidly accelerated toward the nucleus. Classical electromagnetic theory states that the electron loses energy, in the form of a photonic emission, in proportion to the square of the acceleration.

The potential for bremsstrahlung creation increases with increasing particle energy as well as increasing atomic number (Z) of the absorber. The fraction of beta particle energy converted to X-rays is

$$f_{\beta} = 3.5 \times 10^{-4} Z E_{\beta} \quad (7)$$

where E_{β} is the maximum beta energy in MeV. The fraction of electron energy from a monoenergetic electron source, such as an X-ray tube, that is converted to photons is

$$f_e = 10^{-3} Z E_e \quad (8)$$

where E_e is the electron energy in MeV. Shielding must be properly designed to prevent bremsstrahlung (e.g., lead shielding for a strong beta/electron source could do more harm than good). In particular, the use of lower Z material is favored for electron shielding.

Example:

Suppose a 1 mCi Co-60 source is encapsulated in a spherical lead shield such that the thickness is just sufficient to stop all the beta particles emitted by the Co-60. What is the X-ray flux as a function of distance outside the spherical shield?

Solution:

For each decay, cobalt-60 emits a single beta particle at a maximum energy of 0.3179 MeV. Recall that the atomic number of lead (Pb) is 82, such that the fraction of beta energy converted is

$$f_{\beta} = (3.5 \times 10^{-4})(82)(0.3179 \text{ MeV}) = 0.009124$$

The source energy (SE) for the photons should be based on the average beta energy, which is about one-third of the maximum beta energy.

$$SE = f_{\beta} A \frac{1}{3} E_{\beta-\text{max}} = (0.009124)(10^{-3} \text{ Ci})(3.7 \times 10^{10} \frac{\text{decays}}{\text{sec}} / \text{Ci})(\frac{1}{3})(0.3179 \frac{\text{MeV}}{\text{decay}}) = 35,773 \frac{\text{MeV}}{\text{sec}}$$

If the concern for the bremsstrahlung is based upon potential human exposure, the energy received per photon should be assumed to be the maximum possible ($E_{\beta-\text{max}}$); otherwise, it may be assumed that each photon receives the average beta energy. We assume the former situation here, such that the photon generation rate is

$$S_X = \frac{SE}{E_X} = \frac{35,773 \frac{\text{MeV}}{\text{sec}}}{0.3179 \frac{\text{MeV}}{\text{photon}}} = 1.125 \times 10^5 \frac{\text{photons}}{\text{sec}}$$

The latter assumption would have resulted in three times as many photons, but each having one-third the energy. Considering the overall shielded Co-60 to be a point source yields the following relation

$$\phi(r) = \frac{S_X}{4\pi r^2} = \frac{1.125 \times 10^5 \frac{\text{photons}}{\text{sec}}}{4\pi r^2}$$

This does not include the gamma photons from the Co-60 decay.

Heavy Charged Particles (e.g., protons, alphas)

Heavy particles are defined here as those radiations having a rest mass significantly larger than that of an electron. Therefore, heavy charged particles include the proton (p), alpha (α), and other ions. Besides ionization, heavy charged particles can undergo nuclear reactions. Unlike the neutron, these positively charged particles must possess sufficient energy to overcome the repulsive forces of the nucleus for such reactions to occur.

Heavy charged particles lose only a fraction of their energy in collisions with atomic electrons. Heavy charged particles are much more massive than electrons; therefore, such particles are only slightly deflected by the atomic electrons. Consequently, heavy charged particles travel in nearly straight lines through matter. Because of the small, gradual amount of energy transferred from the ion to the absorber

material, the particle passage may be treated as a continuous slowing down process. Near the end of its track, an ion experiences charge exchange between the absorbing material, and picks up electrons until it becomes a neutral atom. The heavy charged particles are the least penetrating radiations for a given energy. Because of the continuous slowing, ions have a much more defined range than the other radiations.

Particle Range Relations

If the range of a heavy particle can be determined in one material (e.g., air), then the Bragg-Kleeman rule[‡] can be applied to determine the range in a second material:

$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1} \sqrt{\frac{M_1}{M_2}} \quad (9)$$

For instance, if air is the medium for which the heavy particle range is known, then substituting the density ($1.23 \times 10^{-3} \text{ g/cm}^3$) and effective atomic mass (14.6 g/mol) for air at 15°C and 1 atm yields:

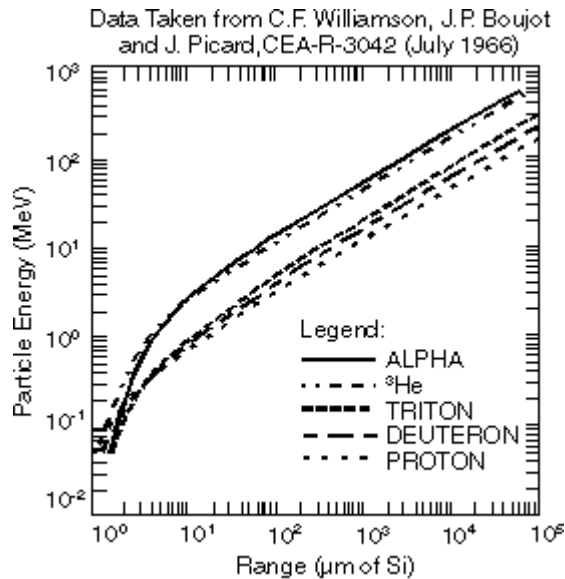
$$R = 3.2 \times 10^{-4} \frac{\sqrt{M}}{\rho} R_{air} \quad (10)$$

For a compound or mixture, the effective atomic mass can be found from

$$\sqrt{M_{eff}} = \sum_i \gamma_i \sqrt{M_i} \quad \text{-- or --} \quad \frac{1}{\sqrt{M_{eff}}} = \sum_i \frac{\omega_i}{\sqrt{M_i}} \quad (11)$$

where ω_i and γ_i are the weight and atomic fractions, respectively, of the i -th constituent.

This figure shows the range (in microns) in silicon of several charged particles, including alpha (α), He-3, triton (T or H-3), deuteron (D or H-2), and the proton (p), as a function of their kinetic energy.



Alpha Particle Range[§]

Alpha particles travel short, straight distances in materials. The range of an alpha particle in air is a few centimeters. The alpha particle range in air (at 15°C and 1 atm) is closely approximated by:

$$R_{air} [\text{cm}] = \begin{cases} 0.56 E_{\alpha} & \text{for } E_{\alpha} < 4 \text{ MeV} \\ 1.24 E_{\alpha} - 2.62 & \text{for } 4 \leq E_{\alpha} < 8 \text{ MeV} \end{cases} \quad (12)$$

[‡] W. H. Bragg and R. Kleeman, *Philosophical Magazine* **10**, 358 (1905).

[§] H. Cember, *Introduction to Health Physics*, 3rd ed., McGraw-Hill, 1996, p. 132.

where E_α is the alpha energy in MeV.

An alpha particle from a natural radionuclide is least penetrating of the radiations examined here, and the “natural” alpha is stopped by a sheet of paper or by dead layer of skin. Since the effective atomic composition of tissue is similar to that of air, the range of alphas in tissue can be computed by applying the Bragg-Kleeman rule to Eq. (12) yielding

$$R_{tiss} = R_{air} \frac{\rho_{air}}{\rho_{tiss}} \approx 1.293 \times 10^{-3} R_{air} \quad (13)$$

where the latter expression in the above equation is based on the density of air at STP ($\rho_{air} = 1.293 \times 10^{-3} \text{ g/cm}^3$) and the density of tissue is assumed to be that of water (1 g/cm^3).

Example:

Determine the range of a 2 MeV alpha particle in air and tissue, and the thickness of aluminum needed to stop the alpha particle.

Solution:

Using the first expression of Eq. (12), the range of the alpha particle in air is computed:

$$R_{air} = (0.56 \frac{\text{cm}}{\text{MeV}})(2 \text{ MeV}) = 1.12 \text{ cm}$$

From Eq. (13), the alpha range in tissue is calculated

$$R_{tissue} = (1.293 \times 10^{-3})(1.12 \text{ cm}) = 0.00145 \text{ cm}$$

The aluminum thickness required to shield the 2 MeV alpha is determined from Eq. (10) using the fact that aluminum has an atomic mass of 26.98 and a density of 2.70 g/cm^3

$$R_{Al} = 3.2 \times 10^{-4} \frac{\sqrt{26.98}}{2.70} (1.12 \text{ cm}) = 0.00069 \text{ cm} = 6.9 \mu\text{m (microns)}$$

Alternatively, a semi-empirical formula for the range in air of an alpha particle is^{††}

$$R_{air} [\text{mm}] = \begin{cases} e^{1.61\sqrt{E_\alpha}} & 1 < E_\alpha \leq 4 \text{ MeV} \\ (0.05 E_\alpha + 2.85) E_\alpha^{3/2} & 4 \leq E_\alpha \leq 15 \text{ MeV} \end{cases} \quad (14)$$

Proton Range

In similar fashion, empirical range relations have been developed for other charged particles. The proton range in air is^{**}

$$R_{air} [\text{m}] = \left(\frac{E_p}{9.3} \right)^{1.8} \quad \text{for } E_p \text{ of a few MeV to 200 MeV} \quad (15)$$

where E_p is the proton energy in MeV.

Bichsel has measured the range of protons in aluminum, and his results can be presented by:^{††}

$$R_{Al} [\mu\text{m}] = \begin{cases} 14.21 E_p^{1.5874} & 1 \text{ MeV} < E_p \leq 2.7 \text{ MeV} \\ \frac{10.5 E_p^2}{0.68 + 0.434 \ln(E_p)} & 2.7 \text{ MeV} \leq E_p \leq 20 \text{ MeV} \end{cases} \quad (16)$$

^{**} E. Segre, *Experimental Nuclear Physics*, vol. 1, 1953.

^{††} N. Tsoulfanidis, *Measurement and Detection of Radiation*, Hemisphere Publishing Corp., 1983, p. 126-128.