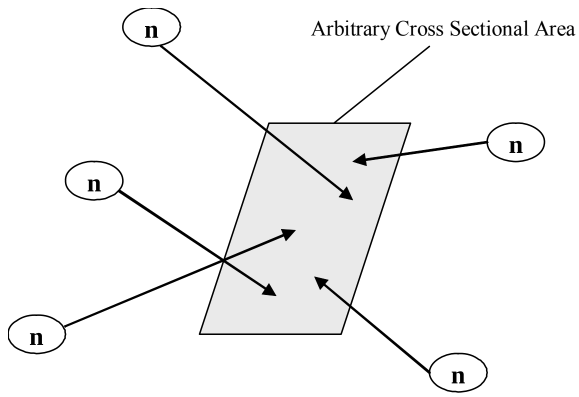
NEUTRON REACTIONS

Neutron Intensity (I) and Flux (ϕ)

When the neutrons are monodirectional, we speak of the neutron intensity (I), but when the neutrons become multi-directional, we change the nomenclature to *flux* (ϕ)

$$I = n\,v \qquad\qquad \phi = n\,v \tag{1}$$

where n is number of neutrons/cm^3 and v is the neutron speed. The beam intensity and flux (ϕ) have units of neutrons/cm^2·sec, which is the number of neutrons crossing through some arbitrary cross-sectional unit area per unit time.



Fluence (Φ)

Fluence is the time-integrated flux (with units of neutrons/cm^2):

$$\Phi = \int \phi(t)\,dt \tag{2}$$

The unit "nv" is an outdated measure of flux in n/cm^2·sec, and similarly, the unit "nvt" is an archaic measure of fluence in n/cm^2.

Reaction (Interaction) Rate

Knowledge of the neutron flux (ϕ) and the material cross sections allows us to compute the rate of interactions. The interaction or reaction rate, RR [reactions/cm^3·sec], is based on total number of neutron-nucleus interactions

$$RR = (n\,v)\,N\,\sigma = \phi\,(N\,\sigma) = \phi\,\Sigma \tag{3}$$

The reaction rate for various types of interactions is found from the appropriate cross section type

$$\begin{aligned} \textit{total reaction rate} &= \Sigma_t\ \phi \\ \textit{fission rate} &= \Sigma_f\ \phi \\ \textit{absorption rate} &= \Sigma_a\ \phi \qquad\qquad \Sigma_a\ \phi \ge \Sigma_f\ \phi \end{aligned} \tag{4}$$

The reaction rate in units of reactions/second can be computed by multiplying RR by the material volume. In similar fashion, the total number of reactions can be determined by using the neutron fluence (Φ), rather than the flux:

$$\textit{Number of reactions} = \Sigma\,\Phi \tag{5}$$

Neutron Attenuation through a target

Because neutrons are uncharged, they have the ability to move long distances through matter. Neutron attenuation calculations are very similar to those for photons:

$$\frac{d\phi}{dx} = -\Sigma_t \, \phi(x) \qquad \Rightarrow \qquad \phi(x) = \phi(0)\, e^{-\Sigma_t x} \tag{6}$$

Fission Rate

The fission rate is

$$\mathit{Fission\ Rate} = \frac{P_{Rx}}{E_R} = V_{Fuel}\ \Sigma_f^{Fuel}\ \phi = V\,(N\,\sigma_f)\,(n\,v) \tag{7}$$

Example: Determine the conversion factor E_R above between fission energy and thermal power.
Solution: Using 200 MeV per fission as the basis

$$E_R = \left(200\,\frac{\text{MeV}}{\text{fission}}\right)\left(\frac{1.602\times10^{-13}\ \text{J}}{\text{MeV}}\right)\left(\frac{\text{W}\cdot\text{sec}}{\text{J}}\right) = 3.20\times10^{-11}\ \frac{\text{W}\cdot\text{sec}}{\text{fission}}$$

$$\frac{1}{E_R} = \left(\frac{\text{fission}}{3.20\times10^{-11}\ \text{W}\cdot\text{sec}}\right)\left(\frac{10^6\ \text{W}}{\text{MW}}\right)\left(\frac{3600\ \text{sec}}{\text{hr}}\right)\left(\frac{24\ \text{hr}}{\text{day}}\right) = 2.70\times10^{21}\ \frac{\text{fissions}}{\text{MW}\cdot\text{day}}$$

Reactor Thermal Power (P_{Rx})

The reactor thermal power can be computed from the fission rate

$$P_{Rx} = (\mathit{Fission\ Rate})\,(\mathit{Energy\ per\ Fission}) = E_R\ V_{Fuel}\ \Sigma_f\ \phi \tag{8}$$

Burnup Rate

The *burnup rate* represents the fuel mass **fissioned** per unit time [g/day].

$$\mathit{Burnup\ Rate} = (\mathit{Fission\ Rate})\,\frac{M_{Fissile}}{N_{Av}} = \frac{P_{Rx}\ M_{Fissile}}{E_R\ N_{Av}} = m_{Fissile}\ \bar{\sigma}_{f,th}\ \phi_T \tag{9}$$

Example: Determine the burnup rate for U-235 to produce 1 MW.
Solution: Using the above formula, the burnup rate is

$$\frac{P\,M_{\text{U-235}}}{E_R\ N_{Av}} = \frac{(1\ \text{MW})(235\ \text{g/mol})(2.70\times10^{21}\ \text{fissions/MW}\cdot\text{day})}{(6.022\times10^{23}\ \text{atoms/mol})} = 1.05\ \text{g/MW}\cdot\text{day}$$

Consumption Rate

The *consumption rate* is the fuel mass converted per unit time [g/day]. The consumption rate includes all absorptions whereas the burnup rate includes only those atoms that are fissioned.

$$\mathit{Consumption\ Rate} = (\mathit{Burnup\ Rate})\,\frac{\sigma_a}{\sigma_f} = (\mathit{Burnup\ Rate})\,(1+\alpha) \tag{10}$$

Example: Determine the consumption rate for U-235 to produce 1 MW.
Solution: For U-235, the thermal neutron absorption and fission cross sections are 680.8 b and 582.2 b, respectively. Using the U-235 burnup rate from the previous example and the above formula, yields a consumption rate of

$$\left(1.05\ \frac{\text{g}}{\text{MW}\cdot\text{day}}\right)\left(\frac{680.8\ \text{b}}{582.2\ \text{b}}\right) = 1.23\ \text{g/MW}\cdot\text{day}$$