NEUTRON CROSS SECTIONS

Cross Sections
The microscopic cross section ($\sigma$) is a property of a given nuclide; $\sigma$ is the probability per nucleus that a neutron in the beam will interact with the nucleus; this probability is expressed in terms of an equivalent area that the neutron "sees." The macroscopic cross section ($\Sigma$) takes into account the number of those nuclides present

$$\Sigma = N \sigma \text{ [cm}^{-1}]$$

The mean free path is $mfp = \lambda = 1/\Sigma$. The microscopic cross section is measured in units of barns (b): 1 barn equals $10^{-24}$ cm$^2 = 10^{-28}$ m$^2$.

Cross Section Hierarchy

Total
   /\     /\      /\       /\
Absorption    Scattering
    /\     /\      /\       /\
Capture       Fission

$\sigma_f = \sigma_s + \sigma_a = \sigma_s + (\sigma_c + \sigma_f)$ \hspace{1cm} where $\sigma_c \approx \sigma_f$

$$\Sigma_f = \Sigma_s + \Sigma_a = \Sigma_s + (\Sigma_c + \Sigma_f)$$

(2)

For mixtures of isotopes and elements, the $\Sigma$'s add. For example

$$\Sigma_{\text{H}_2\text{O}} = \Sigma_{\text{H}} + \Sigma_{\text{O}} = N_{\text{H}} \sigma_{\text{H}} + N_{\text{O}} \sigma_{\text{O}}$$

$$= 2 N_{\text{H}_2\text{O}} \sigma_{\text{H}} + N_{\text{H}_2\text{O}} \sigma_{\text{O}} = N_{\text{H}_2\text{O}} (2 \sigma_{\text{H}} + \sigma_{\text{O}})$$

(3)

$1/v$ Law
For very low neutron energies, many absorption cross sections are $1/v$ due to the fact the nuclear force between the target nucleus and the neutron has a longer time to interact

$$\sigma_{\text{a}} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}} \propto \frac{1}{\sqrt{T}}$$

(4)

Energy dependence of cross sections
- $\sigma_a$ is independent of thermal energy (and temperature)
- $\sigma_a$ ($\sigma_c$ and $\sigma_f$) is energy dependent

$$\frac{\sigma_a(E)}{\sigma_a(0)} = \frac{v_0}{v(E)} = \sqrt{\frac{E_0}{E}} = \sqrt{\frac{T_0}{T}}$$

(5)