

RADIATION COUNTING STATISTICS

Radioactive decay and other nuclear reactions are randomly occurring events, and therefore must be described quantitatively in statistical terms. Not only is there constant change in the activity of a specific sample due to the half-life of the radionuclide, but there is also a fluctuation in the decay rate of a particular sample from one instant to the next due to the random nature of radioactive decay.

The results from counting the radiation(s) from a sample once can be expressed as

$$\text{Counts} = n \pm \sigma = n \pm \sqrt{n} \quad (1)$$

where n is the number of counts and $\sigma = \sqrt{n}$ represents one standard deviation using Poisson statistics.

Because a sample is counted for a specified period of time, the results are reported in units of inverse time, that is, counts per minute (cpm), counts per second (cps), etc. So the equation becomes

$$\text{Count Rate} = \frac{n_s}{T_s} \pm \frac{\sqrt{n}}{T_s} = R_s \pm \sqrt{\frac{R_s}{T_s}} \quad (2)$$

where T_s is the actual sample counting time and $R_s = n_s/T_s$ or counts per unit time, and the subscript s denotes sample. The standard deviation of the sample is an adequate descriptor when sample count rates are large compared to the background radiation but must be modified when counter background is not negligible, *i.e.*, for sample count rates which are in the vicinity of the background count rate. In this situation an additional step is required to calculate the standard deviation of the net count. The step is based on the fact that the variance (the square of standard deviation) of the difference of two independent variables is the sum of their variances. Thus

$$\sigma_s^2 = \sigma_t^2 + \sigma_b^2$$

σ_s^2 = sample variance

$\sigma_t^2 = n_t/T_t^2$ = total (sample plus background) variance

$\sigma_b^2 = n_b/T_b^2$ = background variance

Therefore, the standard deviation for the sample may be expressed as

$$\sigma_s = \sqrt{\sigma_t^2 + \sigma_b^2} = \sqrt{\frac{n_s + n_b}{T_t^2} + \frac{n_b}{T_b^2}} = \sqrt{\frac{R_t}{T_t} + \frac{R_b}{T_b}} \quad (3)$$

where $R_t = n_t/T_t$ = total (gross) count rate

$R_b = n_b/T_b$ = background count rate

The results may now be written as

$$\text{Results} = R_s \pm \sqrt{\frac{R_t}{T_t} + \frac{R_b}{T_b}} \quad (4)$$

where $R_s = R_t - R_b$ = sample (net) count rate

The results in Equation (4) are expressed in counts per minute. To correct cpm to actual activity in the sample, which may be expressed in disintegrations per minute (dpm), we must apply the necessary corrections to the count rate. Some typical corrections include:

1. Counter efficiency = ε
2. Radioactivity recovery from sample preparation procedures = P
3. Self absorption = A
4. Backscatter = B

Applying the necessary corrections will transform the count rate from counts per minute to disintegrations per minute (dpm). Recall that 1 Ci is 3.7×10^{10} Bq. Rewriting Equation (4) to account for errors yields

$$Activity = \frac{R_s \pm \sqrt{\frac{R_t}{T_t} + \frac{R_b}{T_b}}}{\varepsilon A P B} \quad [dpm] \quad (5)$$

Note that not all of the correction factors may be applicable to every counting situation. Notice also that each of the correction factors may have associated with it an error or standard deviation and these must be included in the calculations by means of *propagation of errors*.

The choice of optimal sample and background counting time is determined from:

$$\left. \frac{T_b}{T_t} \right|_{optimal} = \sqrt{\frac{R_b}{R_t}} \quad (6)$$

Since the sample count should be approximately equal to zero (hopefully), then sample and background count rates should be nearly identical, so equal counting times should be spent on both the background and sample count times. For low count rates the *resolving time* is negligible, so *dead time* correction is not required.

Example #1

A detector with a 32% efficiency is used to count a radioactive sample. The sample is counted for 200 mins and yields a total count of 3,050. A 200 min background count is taken immediately thereafter and yields a count rate of 10.0 cpm. The net sample count rate is

$$R_s = \frac{3050 \text{ counts}}{200 \text{ mins}} - 10 \text{ cpm} = 5.25 \text{ cpm}$$

with a sample standard deviation of

$$\sigma_s = \sqrt{\frac{(3050 \text{ counts}/200 \text{ mins})}{200 \text{ min}} + \frac{10 \text{ cpm}}{200 \text{ min}}} = 0.36 \text{ cpm}$$

The activity is then computed as

$$\begin{aligned} Activity &= \frac{5.25 \pm 0.36 \text{ cpm}}{0.32 \text{ counts/disint}} = 16.4 \pm 1.1 \text{ dpm} \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) \left(\frac{1 \text{ Bq}}{1 \text{ dps}} \right) = 0.27 \pm 0.02 \text{ Bq} \\ &= (0.27 \pm 0.02 \text{ Bq}) \left(\frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ Bq}} \right) \left(\frac{10^{12} \text{ pCi}}{1 \text{ Ci}} \right) = 7.4 \pm 0.5 \text{ pCi} \end{aligned}$$

Detection Thresholds

When determining the *detection threshold* of a counting system, the measurement is expressed in terms of background count rates, R_b . The Minimum Detectable Activity (MDA) of a counting system is defined by the National Bureau of Standards as three standard deviations of the background count rate where the sample is counted for the same period of time, hereafter abbreviated MDA (B.S.). Thus, this value is associated with a 99.9% level of confidence that counts greater than MDA (B.S.) represent valid, detectable radioactivity.

$$MDA (B.S.) = 3 \gamma \sigma_b = 3 \gamma \sqrt{\frac{n_b}{T_b^2}} = 3 \gamma \sqrt{\frac{R_b}{T_b}} \quad (7)$$

where

R_b = background count rate (cpm)

T_b = background counting period (min)

γ = correction factor(s) (e.g., $1/\epsilon$) to transform cpm to the desired result (i.e., $\mu\text{Ci/gm}$ or Bq/L).

Confidence Level	Significance Level	Number of Standard Deviations
50%	50%	0.6745
68%	32%	1.0
90%	10%	1.645
95%	5%	1.960
96%	4%	2.00
99%	1%	2.575
99.7%	0.3%	3.00

Threshold detection limits are more rigorously defined using a statistical approach advanced by Altshuler and Pasternack. This theory defines two minimal threshold activities based on the maximum acceptable risks of (1) deciding there is sample activity when there, in fact, is none (Type 1 error; false alarm or false positive), and (2) concluding there is no sample activity when there is some (Type 2 error; missed alarm or false negative). These threshold activities are defined as the

Minimum Significant Activity (MSA) — smallest measurement which is interpreted as meaning there is activity in the sample (for a given level of confidence).

Minimum Detectable True Activity (MDTA) — smallest amount of activity required to be in a sample in order that a measurement can be expected to correctly imply the presence, and correctly quantitatively assay the activity with a predetermined degree of confidence.

Whereas either quantity could be used to represent a threshold detection limit, a distinction is made between both limits since the MSA relates measurements that merit being reported as greater than zero (activity), and the MDTA refers to the minimum amount of actual (true) activity that can be detected with a predetermined level of confidence.

The statistical formulation is categorized into two cases where either (1) the expected value of the background is accurately known or (2) the expected value of the background is not accurately known beforehand. The former case is applicable to most counting instruments. Thus, derivation of the threshold levels for case (1), properly adjusted to data obtained in most counting, yields the expressions

$$MSA = \gamma K_A \sqrt{\frac{R_b}{T_b}} \quad (8)$$

$$MDTA = \gamma \sqrt{\frac{R_b}{T_b}} \left[K_A + K_B \sqrt{1 + \frac{K_A}{\sqrt{R_b T_b}} + \frac{K_B^2}{4 R_b T_b} + \frac{K_A^2}{2 \sqrt{R_b T_b}}} \right] \quad (9)$$

and whenever $\frac{K_A + K_B}{\sqrt{R_b T_b}} \ll 1$ then

$$MDTA \cong \gamma (K_A + K_B) \sqrt{\frac{R_b}{T_b}} \quad (10)$$

K_A = value associated with a predetermined probability of avoiding a Type 1 error (false alarm)
 K_B = value associated with a predetermined probability of avoiding a Type 2 error (missed alarm)

The values for K_A and K_B are given in tables of the normal distribution function, and excerpts are presented in the table below:

Probability of Avoiding a Type 1 and/or Type 2 Error (%)	K_A and/or K_B
99.9	3.00
99.0	2.33
97.5	1.96
95.0	1.64
90.0	1.28

Noting the table, it can be seen that if the probability of avoiding an erroneous activity detection is established to be 99.9%, then $K_A = 3.00$ and the MSA (Eq. 8) is equivalent to the MDA (B.S.).

Example #2

Here the detection thresholds are determined using the data from Example #1. The MDA (B.S.) is

$$MDA (B.S.) = 3 \left(\frac{1 \text{ Bq}/60 \text{ dpm}}{0.32 \text{ counts/disint}} \right) \sqrt{\frac{10 \text{ cpm}}{200 \text{ min}}} = 0.03 \text{ Bq}$$

For this example a confidence level of 97.5% is selected for avoiding both Type 1 and Type 2 errors so that $K_A = K_B = 1.96$. The MSA is

$$MSA = 1.96 \left(\frac{1 \text{ Bq}/60 \text{ dpm}}{0.32 \text{ counts/disint}} \right) \sqrt{\frac{10 \text{ cpm}}{200 \text{ min}}} = 0.02 \text{ Bq}$$

Note that $\frac{1.96 + 1.96}{\sqrt{(10 \text{ cpm})(200 \text{ min})}} = 0.088 \ll 1$ such that the MDTA is

$$MDTA = (1.96 + 1.96) \left(\frac{1 \text{ Bq}/60 \text{ dpm}}{0.32 \text{ counts/disint}} \right) \sqrt{\frac{10 \text{ cpm}}{200 \text{ min}}} = 0.05 \text{ Bq}$$

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Acknowledgement

Much of this document was extracted from a laboratory experiment on "Statistics of Radiation Measurements," which was developed at ASU by the late Professor John W. McKlveen.