TRANSMUTATION

During Irradiation ($t < t_I$):
A "target" with nuclei n_1 is placed in a reactor or similar facility and is exposed to a constant flux of particles, such as neutrons. Some of the target nuclides, n_1, absorb a neutron to form a radionuclide n_2 that here subsequently decays to a stable end product n_3:

$$n_1\,(\textit{target}) \;\rightarrow\; n_2\,(\textit{radioactive}) \;\rightarrow\; n_3\,(\textit{stable})$$

Considering the fundamental rate-of-change equation based on production minus losses, there is both creation and decay of the activation product (n_2) according to:

$$\frac{d n_2}{dt} = \sigma_{a,1}\, n_1(0)\,\phi - \lambda_2\, n_2(t) \tag{1}$$

An assumption is made that the number of target nuclei, n_1, remains constant and that there are few (negligible) neutrons absorbed by the n_2 or n_3 nuclei. Laplace transforms provide a straightforward solution method, especially since there are zero initial conditions (*i.e.*, there are no n_2 or n_3 atoms to begin with):

$$\begin{gathered} s\, n_2(s) - n_2(0) = \frac{\sigma_{a,1}\, n_1(0)\,\phi}{s} - \lambda_2\, n_2(s) \\ n_2(s) = \frac{\sigma_{a,1}\, n_1(0)\,\phi}{s\,(s+\lambda_2)} \end{gathered} \tag{2}$$

Inverse Laplace transforming gives

$$n_2(t) = \frac{\sigma_{a,1}\, n_1(0)\,\phi}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right) \tag{3}$$

The activity of the activation product, n_2, is

$$A_2(t) \equiv \lambda_2\, n_2(t) = \sigma_{a,1}\, n_1(0)\,\phi\,(1 - e^{-\lambda_2 t}) \tag{4}$$

where

- ϕ = the particle (neutron) flux,
- $\sigma_{a,1}$ = the microscopic capture (absorption) cross section of the target nuclide,
- λ_2 = the decay constant of the activation product, and
- t = the time since starting the irradiation.

The buildup of a stable decay product, n_3, may be described by:

$$\frac{d n_3}{dt} = \lambda_2\, n_2(t) \tag{5}$$

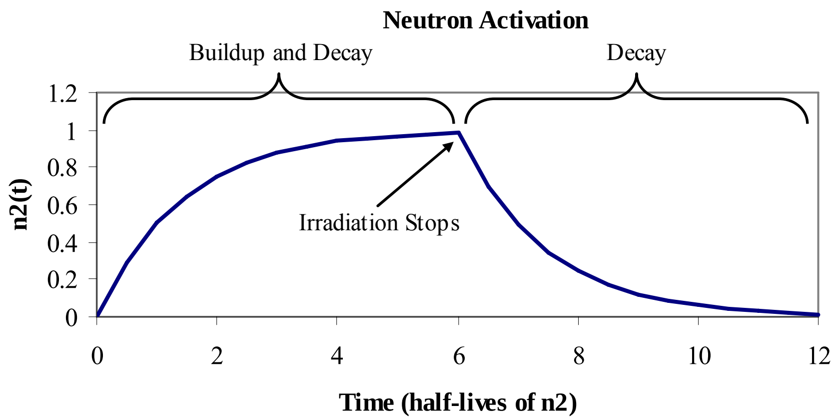
Laplace transforming and substituting the expression for $n_2(s)$ from Equation (2) yields

$$\begin{gathered} s\, n_3(s) - n_3(0) = \lambda_2\, n_2(s) \\ n_3(s) = \frac{\lambda_2\, n_2(s)}{s} = \sigma_{a,1}\, n_1(0)\,\phi\left[\frac{1}{s^2} - \frac{1}{s\,(s+\lambda_2)}\right] \end{gathered} \tag{6}$$

The time domain solution is then

$$n_3(t) = \sigma_{a,1}\, n_1(0)\,\phi\left[t - \frac{1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right)\right] \tag{7}$$

The buildup of the radioactive activation product, $n_2(t)$, during irradiation from t=0 to t=6, where the time scale is normalized to the number of half-lives of n_2. The number of activated nuclei (radionuclides) present at a particular time instant asymptotically approaches $\sigma_{a,1}\, n_1(0)\,\phi / \lambda_2$. After irradiation ceases, decay of n_2 continues.



After Irradiation ($t_I < t$):
Once the irradiation is stopped, there is no longer production of n_2 rather only decay of the activation product n_2 and buildup of the decay product n_3:

$$\begin{aligned} \frac{d n_2}{dt} &= -\lambda_2\, n_2(t) \\ \frac{d n_3}{dt} &= \lambda_2\, n_2(t) \end{aligned} \tag{8}$$

Again, Laplace transforms provide a natural solution, however, the initial condition are non-zero. The "new" initial conditions are determined from the total irradiation time, t_I, (that is, $n_2(t_I)$ and $n_3(t_I)$ are computed from Equations (3) and (7), respectively).

$$\begin{aligned} s\, n_2(s) - n_2(t_I) &= -\lambda_2\, n_2(s) \\ n_2(s) &= \frac{n_2(t_I)}{(s+\lambda_2)} \end{aligned} \tag{9}$$

To retain the same time base, that is keeping $t = 0$ at the same reference point for both the irradiation and decay periods, it is necessary to make a slight adjustment during the inverse Laplace transform operation

$$\begin{aligned} n_2(t') &= n_2(t_I)\, e^{-\lambda_2 t'} \\ n_2(t) &= \left[\frac{\sigma_{a,1}\, n_1(0)\,\phi}{\lambda_2}\left(1 - e^{-\lambda_2 t_I}\right)\right] e^{-\lambda_2 (t - t_I)} \end{aligned} \tag{10}$$

Likewise, the buildup of n_3 may be determined via

$$\begin{aligned} s\, n_3(s) - n_3(t_I) &= \lambda_2\, n_2(s) \\ n_3(s) &= \frac{n_3(t_I)}{s} + \frac{\lambda_2\, n_2(s)}{s} \\ &= \frac{n_3(t_I)}{s} + \frac{\lambda_2\, n_2(t_I)}{s\,(s+\lambda_2)} \end{aligned} \tag{11}$$

Inverse Laplace transforming similarly gives the buildup of n_3

$$n_3(t) = n_3(t_I) + n_2(t_I)\left[1 - e^{-\lambda_2 (t - t_I)}\right] \tag{12}$$

Note how the simple relations in Equations (10) and (12) make physical sense.